Similarity by feature creation: Reexamination of the asymmetry of similarity

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Abstract

We developed a computational model of similarity judgment in problem-solving contexts. The model first attempts to transform an object to another using the knowledge of the domain, the strategy, and the goal. If the transformation succeeds, new feature about transformability is created. A similarity of an object to another is computed, based on the created features. If the model fails to create a new feature, it computes a similarity by feature comparison in the same way as the contrast model. An important prediction of the model is that the asymmetry of similarity judgments is caused by the directionality of the problem-solving skills. We examined the model's prediction. The material was the Tower of Hanoi puzzle. Subjects were required to rate the similarities of one state to the goal as well as those of the goal to a state. In experiment 1, we taught one group of subjects the 'move-pattern strategy' that induced learners to acquire highly directional skills, and compared their judgments with those by naive subjects. The asymmetry was observed only in the judgments by the trained subjects. The second experiment showed that the results of the experiment 1 could not be reduced to the 'prototypicality' of the goal.

Introduction

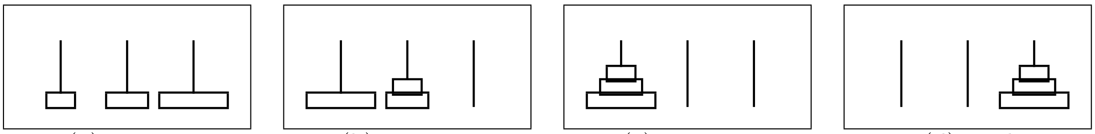
People have an ability to deal with situations to which they are not familiar. This ability is mainly based on the analogical use of past experiences. That is, people retrieve a similar past experience and make use of, or adapt, its solution to the current situation. Various models for analogy have been proposed (Falkenhainer, Gentner, & Forbus, 1989; Hammond, 1990; Holyoak et al. 1989).

There is one important problem concerning analogy, however. As mentioned above, people have to retrieve a similar experience to the current situation in analogy. However, in what sense is a retrieved experience *similar* to the current situation? More generally, how is similarity in problem-solving and learning contexts defined? Feature-based models of similarity such as the contrast model (Tversky, 1977) do not seem to be quite successful. It is because features important in judgment of object-level similarity are not necessarily important in a problem-solving context.

This suggests that, two types of similarities should be distinguished, deep (goal-related) and shallow (surface). Gentner and Forbus (1991) proposed a model, MAC/FAC, that computes similarities of both deep and shallow levels. The MAC/FAC model consists of two stages. While in the MAC stage, computationally cheap matchers act on content vectors of items in LTM, structural examinations are made in order to compute deep similarity in the FAC stage.

Although the MAC/FAC model captures important aspects of human similarity judgment, further steps should be taken to model judgments of similarity. Suzuki, Ohnishi, & Shigemasu (1992) found that people's judgments of similarity are greatly affected by their recognition of the task goal and by knowledge of the domain, using the Tower of Hanoi Puzzle as a material. When subjects did not know the puzzle and were asked to judge the similarity between a state and the goal state, their judgments were based on superficial features shared with both states. In contrast, experts' judgments were dependent on the distance between a state and the goal. For example, while similarity between the states in Figure 1(c) and 1(d) was rated very high by naive subjects, experts' ratings were very low. When given a stimulus set shown in Figure 1(a) and 1(d), patterns of rated similarity were reversed between naives and experts. What happened if similarity of the states is judged by nonexperts who only know the rules of the puzzle? In this case, their judgments were dependent on both the distance and the number of shared features. When a given state was easily transformed to the goal, their judgments were based on the distance. However, they relied on shared features when it is difficult for them to transform a given state to the goal.

These results showed that we should incorporate the goal recognition mechanism and domain knowledge into the model of similarity judgment. Furthermore, these suggest a challenging problem to theories of similarity. That is, not every feature exists prior to the judgment of similarity. Rather some features are created by the recognition of the goal and the knowledge of the domain. Although most of the previous theories have assumed that all the features necessary for the computation of



(a) state 4 (b) state 10 (c) state 20 (d) goal

Figure 1: States of the Tower of Hanoi

similarity be provided externally [1], the assumption is highly dubious especially if one considers similarity judgments in the problem-solving contexts. For example, it is not reasonable to suppose that there are features such as small disk's being movable to the peg C, medium disk's not being movable to the peg B, etc. Rather, these features are created or inferred in the judgment process. Another source of evidence is studies on expert-novice differences. Chi, Feltvitch, & Glaser (1981) showed that novices' sortings of physics problems were based on superficial similarity between the problems, while experts' one on physics principles. These principle-related features seem to be created by experts' knowledge. Without the knowledge, no such features exist in given problems. That is why their sortings were based on superficial similarity between the problems.

Therefore, a model should be developed that creates features by using the goal and the domain knowledge. Although we have already outlined the model in the previous study (Suzuki, Ohnishi, & Shigemasu, 1992), it was not computationally implemented. In this study, we introduce the computational model of Similarity by Feature Creation, hereafter *SFC*, and examine the model's psychological validity in terms of a famous experimental finding, the asymmetry of similarity.

SFC: The Two-Stage Model of Similarity Judgment

Figure 2 depicts the processes of *SFC*. *SFC* consists of two stages: in the first stage, deep similarity is computed, and in the second stage shallow similarity is computed. The inputs are physical description of A and B. In the first stage, *SFC* assumes that queries for the similarity of A to B are those for the transformability of A to B. In other words, B is treated as the goal, while A is treated as another state in the problem space. In the *feature creation* phase, *SFC* first tries to detect differences between A and B. If the differences are detected, the model tests whether A can be transformed to B using the domain knowledge, the goal, and the strategy. The knowledge of the domain consists of the operators. The strategy consists of list of subgoals and their dependency. If the test succeeds, a new feature 'transformable(Goal, Distance)' is created, where 'Goal' is B or the subgoal, and 'Distance' is the distance between 'Goal' and A. If the test fails, another subgoal is retrieved from the subgoal list to test whether the state A can be transformed to the subgoal. This procedure is repeated recursively until the test succeeds.

In the *d-sim* phase, the model computes a deep similarity between a state A and the goal B, based on the newly created feature. If 'Goal' is in the higher branch of the goal dependency tree, the similarity between A and B are rated high. In contrast, the rated similarity is rated low, if 'Distance' is greater.

If the goal is not recognized, or an appropriate subgoal is not retrieved, *SFC* computes a shallow similarity in the second stage. In this stage, similarity is computed in the same way as Tversky's contrast model, using physical description of inputted objects.

We take the Tower of Hanoi as an example to illustrate the performance of *SFC*. Suppose that the inputs are Figure 1(a) (we represent it as [[1],[2],[3]]) and Figure 1(d) ([[],[],[1,2,3]]), and that the sugoal list consists of [[[],[],[1,2,3]], [[-],[-],[2,3]], [[-],[-],[3]]] where '-' represents 'wild card', and the order of elements represents the goal/subgoal hierarchy. Through feature creation stage, a feature 'transformable([[-],[-],[2,3]], 1)' is created, because the state already achieves the subgoal [[-],[-],[3]], and can be transformed to [[-],[-],[2,3]] in one step. Now suppose Figure 1(b) ([[3],[1,2],[]]) is given instead of Figure 1(a). Through feature creation step, a feature 'transformable([[-],[-],[3]], 1)' is created. In the d-sim stage, Figure 1(a) is more similar to Figure 1(d) than the state Figure 1(b) is, because [[-],[-],[2,3]] is closer to the goal than [[-],[-],[3]] is. If feature creation fails, similarity is calculated by comparison of superficial features in s-sim step.

Asymmetry in Similarity Judgments

There is an important issue with which *SFC* has not yet dealt. Tversky (1977) showed that similarity is asymmetric. For example, the judged similarity of North Korea to Red China exceeds the judged similarity of Red China to North Korea.

Although we did not suggest any cognitive mechanisms for the asymmetry of similarity in the previous study, the asymmetry of similarity may also be observed in problem-solving contexts. For example, translating one language to one's mother language is sometimes very different from translating the latter to the former one. Since learners are forced to acquire skills to achieve the

[1] Few exceptions are models based on the case-based Reasoning, such as Kolodner (1989) and Leak (1991).

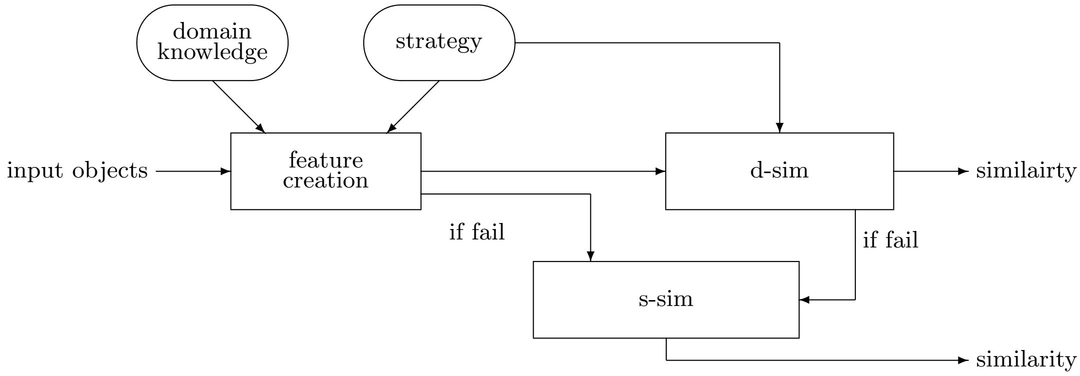


Figure 2: *SFC* model

goal, they may well acquire a set of specialized skills that transform a certain set of fixed initial states to the goal. It is less likely that learners are required to learn reverse operators. In this sense, problem-solving skills are directional.

If the skills are directional, judged similarity of one state to another should sometimes be asymmetric. *SFC* explains this in terms of the subgoal and the domain knowledge. Suppose an expert who is trained in a highly routinized way, where only the transformation of the fixed initial state to the fixed goal is required. Acquired knowledge should be specialized only to transform the initial state to the goal. If he is asked to rate the similarity of the initial state to the goal, his judgment should be a function of distance. Suppose he is asked to rate the similarity of the goal to the initial state. In this case, he recognizes the initial state as the goal and the goal as the initial state. He can also detect the difference between the two states, and tries to transform the goal to the initial state. However, he may not be able to transform the goal to the initial state because the strategy that used to be available is no longer available in this case. Thus, the similarity is computed in the s-sim stage. In these cases, the rated similarity of the initial state to the goal and that of the goal to the initial state are sometimes quite different. We suppose that this is the main source of asymmetry of similarity.

What happens if a naive subject is asked to rate the similarity? *SFC* predicts that the asymmetry of similarity is not observed in such a case. Since the subjects do not recognize the goal or have any operators for transformation, their judgments should be based on the number of shared features. Therefore, judged similarities should not be asymmetric.

In order to explore this hypothesis, we conducted an experiment using the Tower of Hanoi puzzle. Subjects were asked to rate a similarity of a state to the goal and that of the goal to the state. In the training condition, subjects were taught 'move-patern strategy' (Simon, 1975). This strategy can be described as follows: On odd-numbered moves, move smallest disk; On even-numbered moves, move another disk; The smallest disk is always moved from the left to the right to the center to the left peg, and so on. Since this strategy is rather mechanical or rote in a sense that this strategy does not require people to recognize the task structure, it is likely that the subjects acquire routinized skills about the puzzle. Thus, teaching this strategy is likely to enhance the acquisition of highly directional skills of the puzzle.

Experiment 1

Method

Subjects Subjects were 21 undergraduate students. They were randomly assigned to one of the two conditions: the training and control conditions. None of the subjects in the control condition had any prior experience with the Tower of Hanoi puzzle.

Procedure Subjects in the training condition first read instructions that described the rule of the Tower of Hanoi puzzle and the strategy to solve it. The strategy taught to subjects was 'move-pattern strategy', as described earlier. Then subjects proceeded to the training phase. In this phase, they were given the three-disk puzzle with a fixed initial state where all disks were on the leftmost peg and asked to move all the disks to the rightmost peg. After subjects could solve it in six successive sessions, they proceeded to the rating phase. In this phase, subjects were asked to judge the similarity between the goal and the other states. Of the total of 52 pairs, a half asked to judge the similarity of the goal to another state and a remaining half asked that of a state to the goal. Subjects were asked to rate how similar the states were, and to circle '7' if the pairs were very similar, '1' if they were very dissimilar, and other numbers for the intermediary degrees of similarity. Subjects in the control condition skipped the instructions and training phase, and proceeded directly to the rating phase.

Results and Discussion

Before analyzing the asymmetry, we examined whether the similarity judgments made by the subjects were based on both domain knowledge and the goal.

The mean regression coefficients between distance and the rated similarity were -0.31 in the training condition and 0.05 in the control condition. The difference between group was significant ($t(11.5) = 4.75; p < 0.01$). This confirmed that the subjects in the training condition incorporated the goal and knowledge of the domain into similarity judgments.

The degree of the asymmetry was defined as $|sim(X, G) - sim(G, X)|$, where G and X represented the goal and one of the states, respectively. The mean degree of asymmetry in the training condition was 0.89, while that in the control condition was 0.41 ($t(19) = -3.50; p < 0.01$).

Our hypothesis was supported by the results that the degrees of asymmetry of similarity judged by the trained subjects exceeded those judged by the subjects in the control condition. The reason the subjects in the training condition judged similarities asymmetrically was due to the fact that the subjects in the training condition acquired the directional skills specialized to achieve the fixed goal.

Although we concluded the difference between the two conditions was attributed to whether subjects recognized the goal and had appropriate operators, there might be an alternative interpretation. Subjects in the training condition frequently observed the goal state in the training session. Every time they practiced, they had to keep the goal state in mind. This might leads features of the goal to be more salient. In other words, the goal might be a 'prototype' state, just as China is more prototypical than North Korea in a Tversky's example. If so, observed differences between the two conditions could not be attributed to the recognition of the goal and knowledge for feature creation.

Experiment 2

In order to examine whether the observed differences between the two conditions merely reflect the salience of the features of the goal state, we conducted the second experiment.

One way to examine this possibility is to compare the ratings of the subjects in the training condition in Experiment 1 with those of other subjects who are trained to acquire flexible (non-directional) skills. If two groups of subjects received the same amount of training but the degree of asymmetry is different, the results cannot be explained by prototypicality of the goal states. There are several different strategies to solve the Tower of Hanoi puzzle (Simon, 1975). One of the strategies is 'perceptual strategy.' It can be described as follows: To construct the tower of disks on the right peg, the largest disk must be placed on the right peg first, the next largest, and so on; to move the largest disk on the right peg, the others must be placed on the center peg. Unlike the move-pattern strategy, this strategy leads subjects to construct subgoals, which leads them to motive the task structure. In this sense, the skills acquired by this strategy are flexible. If so, the degree of asymmetry should be less than those in the fixed trained condition.

Method

Subjects Subjects were 15 undergraduate students. They were randomly assigned to one of the two conditions, the MPS (move-pattern strategy) or the PS (perceptual strategy) conditions.

Procedure The procedure and the material for the MPS condition were identical to the training condition in the Experiment 1. Except that the perceptual strategy was taught to the subjects in the PS condition, the procedure and the material for the PS condition were identical to the MPS condition. Subjects in the both conditions were asked to solve the puzzle six times in the training session. The initial state in this session was fixed so that all the disks were placed in the leftmost peg. The goal was also fixed so that all the disks were placed in the rightmost peg.

Results

The mean degree of asymmetry in the MPS condition was 0.84, while that in the PS condition was 0.74. The difference of the asymmetry between two conditions was not significant ($t(10.8) = 0.8173; p = 0.43$). Subjects in the both conditions judged the similarity asymmetrically. This might reflect that the degree of expertise of the subjects in the PS condition was less than we initially expected.

Thus, we restrict our analysis to those pairs where each state was on the optimal solution path. Since the three-disk puzzle was quite easy for the subjects in this experiment, they rarely went out of the optimal solution path. Thus, the subjects in the PS condition might acquire non-directional operators for states at least on this path. If so, these subjects might judge the similarity symmetrically.

The mean degree of asymmetry of the pairs on the optimal solution path in the MPS condition was 1.03, while that in the PS condition was 0.43. The difference between two conditions was significant ($t(13) = -2.70; p < 0.05$).

General Discussion

We hypothesized that some features were created by the goal and knowledge, and that the asymmetry of similarity was attributed to the directionality of acquired skills. The results of the experiments confirmed the hypothesis. The asymmetry of similarity took place in a problem-solving context. When a subject recognized the goal and had directional skills that were acquired through a routinized training, similarity of one state to the goal was judged to be higher than that of the goal to the state. This is because acquired skills are so directional that can be applied only to a small subset of possible states. In addition, the Experiment 2 showed that the phenomenon could not be reduced to the prototypicality of the goal. Even when the goal state was observed more frequently than the others, subjects with a flexible strategy did not make asymmetric judgments. This is because such subjects acquire knowledge flexible enough to deal with unfamiliar situations.

SFC is in a sense similar to the 'transformation structure model' (Imai, 1977). According to his model, people judge that the pattern A is similar to the pattern B when A can be transformed to B. A degree of similarity is defined as the number of operators that transform one item to another. However, his model does not provide any explanation for the asymmetry of similarity as well as the expert-novice differences found in the previous study (Suzuki, Ohnishi, & Shigemasu, 1992).

By incorporating the goal and knowledge into the model of similarity, *SFC* has several implications for the study of learning. It is well-known that experts attend to structural aspects of problems, whereas novices do to superficial ones. According to *SFC*, this difference is due to whether one can create relevant features. Novices lack appropriate knowledge of the domain that enables them to create features structurally organized by the principles of the domain. This leads them to the reliance on superficial features.

As Hatano (1986) suggested, there are two types of experts, routine experts and adaptive experts. While a routine expert has highly specialized skills to solve a fixed set of possible problems in the domain, an adaptive one has skills that can be transferred to new situations. This is explained by the directionality of knowledge. A routine expert acquires knowledge through the training similar to the MPS condition in the Experiment 2. In such a training, acquired skills are highly directional to the goal. If so, such an expert may find it very difficult to create goal-relevant features in unfamiliar situations. In contrast, experts with generalizable skills may find it less difficult to do it.

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