Bridging the Conceptual Gap

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Abstract

This paper claims that, contrary to the " Theory-oriented" approach to cognitive development and instruction, children's informal concepts play important roles in learning, and reports two cases that support the constructivist view of learning. It is widely believed that children's knowledge about various domains is organized into coherent systems, i.e., theories. Although this approach provides a new perspective on knowledge organization, too much emphasis on the conceptual difference makes the interaction of prior knowledge and learning materials impossible. Without the interaction, learned rules remained uninterpreted. Consequently they can be applied only to a restricted set of problems. A case from mathematics revealed that students' informal concept of concentration can be bridged to the formal one, by rewording quantitative terms in problems with qualitative terms. A case from physics showed that by combining fragmentary understandings, students could acquire the concept of force decomposition which is difficult to learn by formal instruction. Finally, instructional techniques are proposed that make use of informal concepts as a partial base analog to enrich students' understanding.

Introduction

Recent studies on cognitive development have revealed that even young children have a great deal of knowledge in various domains such as physics, biology, mathematics, psychology, etc. This finding leads not a few researchers to claim that children's knowledge of the domain is not merely an incomplete version of the adults' one, but that it is organized into a coherent system, " Theory."

What does it mean by theories? Wellman and Gelman (1992) suggested that theories involve the distinctive ontology, causal mechanism, and coherence. An observed phenomenon is reduced to presupposed entities by the ontology of the theory, while the behaviors of these entities are explained or predicted by the causal mechanism of the theory.

Some researchers claim that children have their own theories that are fundamentally different from adults' ones. Seminal work by Carey gave evidence that children's theory of biology is organized by the naive psychology (Carey, 1985). That is, people are a prototype of living things and any biological judgments are made in terms of the similarity to the prototype.

The problem here is how the conceptual change occurs, the shift from one theory to another. Unfortunately few researchers have made it clear. One of the reason is that the conceptual change is logically very difficult. Strictly speaking, two different theories are incommensurable in the sense that certain terms in one theory do not correspond to any terms in another.

Cases showing incommensurability can be found abundantly in the history of science. By comparing the modern thermal dynamics with the preceding one, Wiser and Carey (1983) showed that scientists in the

17th century presumed the existence of hot/cold particles which have no correspondents in the modern theory and explained thermal equilibrium in terms of the emission of these particles. Furthermore, counter-evidence is not effective for the theory change. Counter-evidence is sometimes resolved by adding new elements to the existing theory. For example, the fact that burned objects become heavier appears to be counter-evidence for the " phlogiston" theory of combustion. However, scientists at this age thought that phlogiston had " lightness." Since phlogiston was supposed to be consumed during the combustion, lightness lost. That was how the scientists explained why burned objects become heavier after the combustion.

If the same were true for the conceptual change in students, it would follow that students cannot acquire a new theory. Actually, in some domains, concepts such as force, natural selection, heat, etc., have been found to be quite difficult to learn. Chi (1993) claimed that these are members of the " acausal interaction" category in the ontological tree. This category involves many characteristics that distinguish itself from the others. Members of this category do not have any causes, the beginning, nor the end and proceed according to constraining relations among their components. These characteristics make the understanding of this category quite difficult.

Chi further claimed that the main source of the misunderstanding lies in the fact that people try to assimilate these concepts to categories familiar to them. Most of the misunderstandings in physics are caused by making force belong to the matter category. For example, naive people hold that force is a kind of impetus and can be used up. This view leads them to think that motion is initiated by agent's giving force, that force decreases because it is consumed during motion, and that motion ends when force is used up.

The implication of Chi's claim for the instruction is to teach the existence and characteristics of the new category, and to prevent them from being related to others, at least, at the early stage of learning.

Problems of Variable Interpretation

Some sorts of theory-like knowledge should be necessary to take into account the early competence and difficulties in learning. However, the radical theory-oriented approach leads us to the problem of incommensurability.

Furthermore, it is highly dubious that one can keep informal concepts out in formal instruction, for the following reason. When learning formal rules, learners are expected not only to memorize them but also to apply them to new situations. In order to do this, it is necessary for them to interpret variables in the rule. In other words, learners should specify what a variable refers to or what class a variable represents. When applying a rule, learners also have to interpret information in a given problem and map it to corresponding variable in the rule. Thus, interpretation is a key to learning formal rules. Interpretation is especially crucial in learning physics rules. Unlike rules in logic, physics rules involve variables that must be interpreted as specific classes of entities in the world (Suzuki, 1994).

A proper interpretation of variables requires full understanding of the domain. However, it is the end product of learning, not the prerequisite for it. Thus, learners sometimes give up the interpretation of variables. However, without interpretation, variables in the formal rule are mere " symbols" in the sense that they do not have any referents in the real world. In consequence, learners who do not interpret variables exhibit strong context specificity. They can apply the rule only to a very limited set of problems that are sufficiently similar to those used in the practice (diSessa, 1983; Reed, 1993). For example, most students can solve a^2-b^2, when they learn an algebraic such as x^2-y^2. However, more than half of the students cannot solve $4x^2-y^2$. Even fewer students can solve x^4-y^4. This indicates that students interpret the variable x and y in the original formula as letters and the exponent as a mere superscript.

What knowledge is responsible for interpretation if one does not have knowledge of the domain to be

learned? One of the knowledge structures for interpretation should be our prior knowledge of the domain that has been acquired through everyday experience.

Informal concepts, in contrast to formal ones, are " grounded," because they are originally developed to deal with real world problems. In describing p-prims, diSessa (1993) characterized the nature of informal concepts. According to his description, " they are ready schemata in terms of which one sees and explains the world"(pp. 112).

Figure 1 illustrates the relationship between informal concepts, formal concepts, and the real world. Although the formal concept potentially has wide applicability (the dotted circle), it covers only a small subset of the real world problems (the black circle) at the early period of learning. In contrast, the informal concept has greater applicability compared with the formal counterpart at the initial stage of learning (the shadowed circle). If the formal concept is related to a corresponding informal concept, it acquires greater applicability to real world problems, because it can make use of the interface of informal concept to the real world.
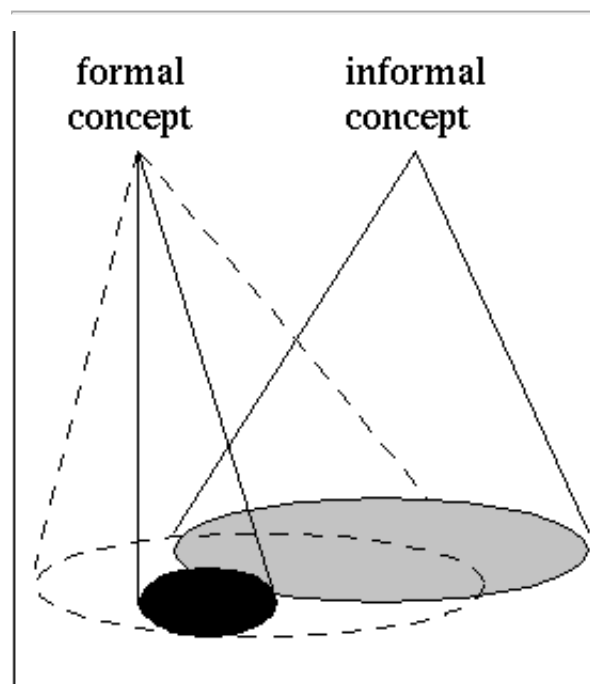

Figure 1:
The relationship between formal and informal concepts. The dotted circle represents a set of problems to which a formal concept can potentially be applied. The black circle represents a set of problems to which a formal concept at the early stage of learning can actually be applied. The shadowed circle represents a set of problems to which an informal concept can be applied.

The theory-oriented approach has another problem, the status of informal concepts after instruction. If instruction proceeds without referring to the related informal concepts, it follows that two kinds of concepts coexist in learners' knowledge base. If so, it is highly likely that students are at a loss as to which concepts should be used. To make the matters worse, informal knowledge has been reinforced repeatedly, because it is used very often and gives approximately the right answer in many occasions (Holland et al., 1986). Consequently, the informal knowledge is more likely to be triggered than the formal ones. Even when the use of the informal concepts is explicitly inhibited, students do not know why they have to use the formal concepts rather than informal ones.

Such learning is far from proper understanding and appreciation of scientific concepts. Examining the utilities of theoretical constructs and clarifying the relationships between them are crucial parts of scientific activity.

Utilizing Informal Concepts in Formal Instruction

The discussion above shows that informal concepts do and should affect the process of learning formal concepts, and that relating formal and informal concepts enables learners to achieve deeper understanding, and prevents them from the mechanical memorization.

Actually, there are domains that children spontaneously use their informal concepts to predict the behaviors of unfamiliar things. Biology is a good example of it. A series of experiments conducted by Inagaki and Hatano showed that even young children can make use of prior knowledge about humans in making predictions about behaviors of other living things (Inagaki & Hatano, 1987, 1991). Additionally, they do not use the prior knowledge haphazardly. Their use is constrained by similarity of a living thing in question to humans. Another source of evidence comes from a series of studies by Cheng and her colleagues (Cheng, et al., 1986). In their previous study (Cheng & Holyoak, 1985), they revealed that people spontaneously acquire a schema that is functionally equivalent to the rules of conditional reasoning when problems involve permission conditions of someone's action. They used this schema as a base analog to teach the logical conditional and gave the Wason's selection tasks as a posttest. The result is that subjects' performance dramatically improved.

However, one might argue that these are exceptional cases. In other domains, informal concepts cannot be used directly in formal instruction. There are at least two cases that should be distinguished. The first case is that although learners have relevant informal concepts, they fail to use them spontaneously. This will be discussed in the next section by analyzing students' informal knowledge of concentration. The second case is that useful informal concepts appear not to be available. This will be discussed in the third section, by investigating students' knowledge of force decomposition.

How to Activate Informal Knowledge: A Case from Mathematics

A consistent finding in research on learning and instruction is that people have considerable difficulties in making use of prior knowledge in situations where it is relevant (Gick & Holyoak, 1980; Reed, 1993). In other words, human knowledge is so context-bound that it cannot be used in different contexts.

I requested more than a hundred of sixth graders to solve the following problem:

> When 100g of 50% orange juice is mixed with 500g of 100% orange juice, what is the percentage of the mixed juice? Estimate the percentage.

30 to 40% of them added the two concentration values. Moreover, about 80% of the answers were higher than the pure orange juice (100%). According to the results, they appear not to understand non-additivity of intensive quantities.

The results seem to be in line with Chi's claim. Since concentration is a member of intensive quantity, it must not be added each other. However, students at this age are so familiar with extensive quantities (weight, length, etc.) that they tend to assimilate concentration to this type of quantity, though concentration is a member of intensive quantity.

However, a posttest interview showed that it is not always the case. After the experiment, I asked one student who had failed on the above problem whether she could make pure orange juice by mixing two cans of 50% orange juice. Her answer was " absolutely not." This anecdote suggests that they might understand the non-additivity principle, but that they could not apply the principle to the above problems.

Why could not they use it? It is often the case that two congruent knowledge structures originated from different experiences have distinct views for the world. By observing his daughter's calculation of multidigit subtraction, Lawler (1981) found that she had several microviews. Each microview involved a distinct view for problems and encoded information differently. For example, the money microview encoded as four quarters and a penny, while the decadal microview decomposed the same problems as tens and ones.

The same might be true for students' understanding of non-additivity, because concentration is termed differently in the real world and classroom. For example, in everyday life, concentration is expressed by a qualitative term, such as sweeter, less sweet, salty or less salty. Concentration is expressed numerically in mathematics, on the other hand. If it is true, the everyday concept of concentration is likely to be triggered when such qualitative terms are used, rather than quantitative terms.

In order to test this hypothesis, 35 sixth graders were examined in the following three conditions (Suzuki, 1987). In the mathematical condition, students were given juice mixing problems where concentrations were expressed numerically. The form of the problem was as follows: " When Xg of x% juice was mixed with Yg of y% juice, estimate the concentration of the resultant juice." Answers were judged to be correct when their estimates were between the concentrations of two original juices. This condition was expected to facilitate quantitative encoding of the concentrations. In the real condition, real juices were presented, but the problems had the same verbal form as in the mathematical condition. In the third condition, the qualitative condition, the problems had the following form: When very sweet tea with X spoonfuls of sugar was mixed with less sweet tea with Y spoonfuls of sugar, did the mixed tea get sweeter, less sweet or the same as the original? This condition was expected to encourage students to represent concentrations qualitatively.

Two types of problems were used, EQUAL and DIFFERENT. In the EQUAL problems, two juices which had the same concentration values were used in the mathematical and real conditions. In the qualitative condition, two cups of tea with the same spoonfuls of sugar were used. While in the mathematical and real conditions 50 and 100% orange juices were used as DIFFERENT problems, two cups of tea with different spoonfuls sugar were used in the qualitative condition.

The results presented in Table 1 clearly show that sixth graders properly understood the non-additivity principle. When the form of the problems was modified so as to trigger their informal concept, the proportion of the correct estimates dramatically increased. Hence, their difficulties lie not in the lack of understanding but in the failure of applying their informal knowledge to problems with the mathematical form. These results show that the informal concept of non-additivity is, at least, functionally equivalent to the formal counterparts and indicate that information that can be encoded is different between the two.

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Table 1: The number of the correct estimates on the juice--mixing
  problems. Numbers in parentheses represent the proportions.
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	Equal 50%+50%	Different 50%+100%
Math	9(25.7)	13(37.1)
Real	17(48.6)	20(57.1)
Qualitative	33(94.3)	26(74.3)

It first appears that students assimilated the unfamiliar quantity into a wrong category. However, a careful analysis of the encoding mechanism revealed that students have informal knowledge functionally equivalent to the formal counterpart.

Coordinating Fragments: A Case from Physics

However, one might still argue that the above example is a lucky case, because entities in the informal concept of concentration can be translated directly to the formal one. In other domains such as physics, it is often impossible to find informal concepts that are directly mapped to the formal counterparts.

For example, McCloskey and his colleagues showed that people sometimes predict the movement of a falling body correctly, against the notorious straight-down belief (McCloskey, et al., 1983). However, such a prediction is made only when the falling object moved autonomously before falling down. If the

object is carried by something else, they cannot make the correct prediction. Since the autonomy of the movement has nothing to do with formal physics, it is impossible to find a formal concept that can be mapped to the informal one.

The notion of " distributed encoding," proposed by diSessa, is useful to deal with such an incompatibility between formal and informal concepts (diSessa, 1993). He described that a physical law was distributed over many p-prims which play some roles in understanding and using the law.

This means that some components of a physics law are shared by one informal concept, and other components by other concepts. No single informal concept corresponds to a physics law, but coordinating these concepts could promote proper understanding of physics laws.

Suzuki (1990) explored this possibility. In his studies, students were required to answer the problem shown in Figure 2(a). This figure illustrates a situation where two individuals are trying to hold a 10kg bag. The question was how much force was required for an individual to keep the bag held. To solve this problem, one must decompose the downward vector into two along with the strings. One way to decompose it is to construct a parallelogram whose diagonal line is a reverse vector of the downward one (as shown in Figure 2(b)).
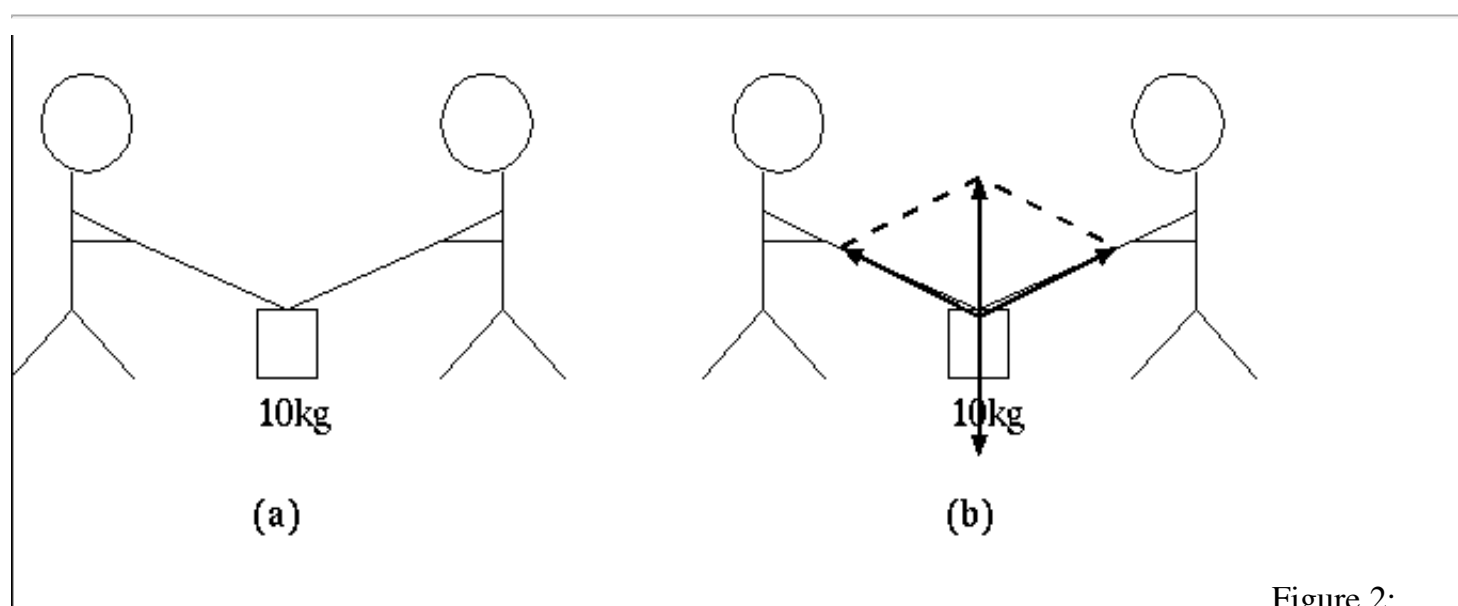

Figure 2:
(a) The problem used in theexperiment. This figure illustrates that two men are trying to keep a 10kg bag held, whose two straps construct an angle of 150 degree. Subjects were required to estimate how much force is required for an individual. (b) The figure illustrates the parallelogram construction method for the decomposition of force.

While this procedure is rather simple, it sometimes produces incredible results. When the angle of two strings is 120 degree, the force required for an individual is as much as the downward force. It means that each individual has to exert 10kg force to keep the 10kg bag held. The reason is that two men are pulling against each other as well as holding the bag. In other words, horizontal vectors are involved in holding the bag.

A preliminary study revealed that force decomposition problems were quite difficult even for university students. More than 80 of students' estimates were less than 10kg (mostly, 6 to 8kg).

To investigate sources of their difficulty, we conducted three instructional experiments. The first experiment examined a possibility that students merely forgot the rule of force decomposition or they could not realize the applicability of the force decomposition rule to the above problem. Thus, we explained the notion of force decomposition and taught the parallelogram construction method, by using an example problem. The second experiment explored a possibility that students' difficulty lies in their

ignorance of conceptual aspects of force decomposition, that is, the angle dependency. Thus, we emphasized that the amount of the horizontal force was dependent on the angle along which the downward force was decomposed, by using multiple examples. In the third experiment, we taught subjects the procedures of parallelogram construction in a step-by-step way and gave three example problems to facilitate proceduralization of the rule.

However, none of the instruction facilitate subjects' performance, as shown in Table 2. They usually neglected the existence of horizontal forces and simply divided downward force by the number of holders, then used the anchoring strategy to adjust their judgments.

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Table 2.: The results of the three instructional experiments.  The
     first column represents subjects' estimates of the required force
     (X) for an individual.
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	Exp.1 Reminding	Exp.2 Angle Dependency	Exp.3 Proceduralization
X < 5kg	4.4	14.9	4.3
X = 5kg	14.8	12.8	47.8
5kg < X < 10kg	58.8	51.1	22.8
X > 10kg	22.1	21.3	25.0

However, it is not always the case. Another study revealed that students could attend to the horizontal forces when force decomposition was instantiated in a rope-pulling contest situation. Most of the students' answers exceeded the weight of the object to be held.

These results show that people have two schemas concerning force decomposition. One of them, the collaboration schema, encodes vertical forces and the number of agents (holders) and computes required forces, by dividing the former by the latter. This schema generates estimate of required force that is less than the weight of the object being held. Another, the competition schema, mainly encodes horizontal forces and computes relatively greater amount of forces.

The results also indicate a possibility that informal concepts are fragmentary and that the components of the formal concept of force decomposition are distributed in two schemas. If so, it is expected that by coordinating two schemas (fragments), students could realize the existence of horizontal and vertical forces and understand the formal concept of force decomposition.

In order to enhance students' coordination of the two schemas, we explicitly mentioned the presence of two kinds of forces. First, by showing a figure of a rope-pulling situation, subjects were told that horizontal forces were present because two individuals were pulling against each other. Next, subjects were asked to notice the fact that the these two were holding a 2kg object, and were told that vertical force exists because of their holding the object. Finally, the force decomposition rule, the parallelogram construction method, was introduced so that two kinds of forces were composed.

The results showed that the proportion of the correct estimates was more than 50%. In contrast, that of the control group who had not been given such instruction was only 8%. These results indicate that fragments of understanding can be organized into coherent one by providing subjects with a situation where both the fragments are involved. The encoding of the force decomposition rule here seems to be distributed across the competition and collaboration schemas (diSessa, 1993).

Discussion

The theory-oriented approach provides a new perspective for the study of learning and cognitive development. It has revealed that children's knowledge constitutes a coherent system which provides efficient constraints in learning in informationally problematic environments.

However, its instructional application, especially when the incommensurability is emphasized, is not very much attractive and in a sense wrong. It is not attractive because this approach explains theory change only in terms of the replacement of an old theory with a new one. Thus, the issue of reorganization of knowledge remains intact. It is in a sense wrong because this approach misses what is shared between formal and informal concepts. It also misses the roles that informal concepts play in understanding and applying formal ones.

The results of the experiments revealed that informal concepts can be building blocks for understanding and applying formal concepts. Furthermore, it provides useful techniques for teachers to teach formal concepts, without isolating them.

Bridging informal concepts to formal ones

Sometimes it is difficult for students to use their informal concepts in classroom problem solving contexts. It is because the encoding functions are different between the two types of concepts.

As shown in the experiment, the informal knowledge of the non-additivity principle articulates the world qualitatively. If a problem involved qualitative terms, the informal knowledge was likely to be triggered and gave approximately the right answer. However, when concentration values were presented numerically, the knowledge was less likely to be activated.

In this case, teachers can help students by translating quantitative terms to qualitative ones. For example, it might be helpful to tell students that 100% juice is thicker than 50% juice. Although it is less likely that the informal concept enables students to invent the proper mathematical solution, it monitors the problem-solving processes and the solution by constraining the possible combination of mathematical operations (Suzuki, 1987).

More generally, educators as well as researchers should be very careful in analyzing students' errors. Their erroneous answers do not always reflect lack of understanding. Rather, these answers may be generated by a sort of repair heuristics triggered when their informal knowledge can not encode the information in a given problem. Knowledge spontaneously acquired through everyday experience often has different encoding functions from those of formal concepts.

Coordinating fragmentary understandings

Informal concepts are sometimes fragmentary, and components of a formal concept are distributed across different informal concepts. As shown in the experiments on force decomposition, the collaboration schema encodes the vertical force and divides it by the number of agents. However, this schema cannot deal with horizontal forces. On the other hand, the competition schema encodes the horizontal forces, but cannot encode and process the vertical force.

In these cases, it is necessary for teachers to look for or devise a situation in which fragmentary understandings are naturally involved. Then, they can explain which components of the informal knowledge are relevant for which aspects of the situation, by emphasizing the examination of the situation from multiple perspectives.

This technique can be regarded as an instructional application of diSessa's " Knowledge-in-Peace" theory. While emphasizing to view a situation from multiple perspectives changes the cuing priorities of the two schemas, presenting the formal concept in terms of two schemas leads its distributed encoding.

References

Carey, S. (1985) *Conceptual Change in Childhood*. Cambridge, MIT.

Cheng, P. W., & Holyoak, K. J. (1985) Pragmatic reasoning schemas. *Cognitive Psychology*, 17, 391-416.

Cheng, P. W., Holyoak, K. J., Nisbett, R. E., & Oliver, L. M. (1986) Pragmatic versus syntactic approaches to training deductive reasoning. *Cognitive Psychology*, 18, 293-328.

Chi, M. T. H. (1993) Barriers to conceptual change in learning science concepts: A theoretical conjecture. *Proceedings of the Fifteenth Annual Conference of Cognitive Science Society*.

diSessa, A. (1983) Unlearning Aristotelian physics: A Study of knowledge-based learning, *Cognitive Science*, 6, 33-75.

diSessa, A. (1993) Toward an epistemology of physics. *Cognition Instruction*, 10, 105-227.

Gick, M. L. & Holyoak, K. J. (1980) Analogical problem solving. *Cognitive Psychology*, 12, 306-355.

Holland, J. H., Holyoak, K. J., Nisbett, R. E., & Thagard, P. R. (1986) *Induction: Processes of Inference, Learning, and Discovery*. Cambridge, MA: MIT Press

Inagaki, K. & Hatano, G. (1987) Young children's spontaneous personification as analogy. *Child Development*, 58, 1013-1020.

Inagaki, K. & Hatano, G. (1991) Constrained person analogy in young children's biological inference. *Cognitive Development*, 6, 219-231.

Lawler, R. W. (1981) The progressive construction of mind. *Cognitive Science*, 5, 1-30.

McCloskey, M., Washburn, A., & Felch, L. (1983) Intuitive physics: The straight-down beliefs and its origin. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 9, 636-649.

Reed, S. K. (1993) A schema-based theory of transfer. In D. K. Detterman R. J. Sternberg (Eds.) *Transfer on Trial*. Norwood, NJ: Ablex.

Suzuki, H. (1987) The roles of formal and informal knowledge in mathematical problem-solving. Paper presented at ninth biennial meetings of the International Society for the Study of Behavioural Development. July, Tokyo

Suzuki, H. (1990) Pragmatically-based representation as a source of naive concepts in physics. Paper presented at the Symposium on " Use of students' informal knowledge as the basis of instruction" at 22nd International Congress of Applied Psychology, Kyoto, July.

Suzuki, H. (1994) The centrality of analogy in knowledge acquisition in instructional contexts. *Human Development*, 37, 207-219.

Wellman, H. M. & Gelman, S. A. (1992). Cognitive development: Foundational theories of core domains. *Annual Review of Psychology*, 43, 337-375.

Wiser, M. & Carey, S. (1983) When heat and temperature were one. In D. Gentner A. L. Stevens (Eds.) *Mental Models*. Hillsdale, NJ: Erlbaum.